

Students Model and Represent the Handshakes Problem

The Pacific Northwest school district recently received a large influx of immigrant newcomers, many of whom are also refugees or asylees. For many years, to meet the need of their existing newcomers, the district held a summer language program on the campus of a local college. This summer language program served students from grades 2-11, and in the pre-pandemic past also served their family members and caregivers. University students participated in the program each summer, including as classroom assistants and bus attendants. In the summer of 2023, the district invited researchers developing *Reimagining and Amplifying Mathematics Participation, Understanding, and Practices* (RAMPUP) as a summer bridge field test.

The first module of RAMPUP was about patterns of growth and change. Students had already explored growing patterns of shapes which they were able to describe, count, analyze, and extend. In the second half of the patterns module, students explored other problems connecting real-world scenarios with numerical sequences that follow a pattern. On day 4 of the summer, the students began to explore the “handshakes problem” which is well-known in mathematics education:

If five people shake hands with each other, how many handshakes will they make?

As you read the vignette, consider the following:

- What do you notice in the students’ interactions and the teacher’s interventions?
- What elements of formative assessment practices do you see in action:
 - Setting clear conceptual goals and shared success criteria
 - Eliciting evidence of student thinking through talk
 - Gauging student performances and offering feedback to bridge the gap to desired aims
 - Engaging students in peer- and self-assessment processes

(Heritage, Walqui, & Linquanti, 2015)

The teacher frames the question in the context of how people can greet each and introduces the scenario to be explored.

Teacher: How do you greet in your culture?

Nasir: [joking, smiling, gesturing] Hugs.

Teacher: If everyone shook hands, how many handshakes happened? If everybody shook hands, how many handshakes happened?

Students counted up the people in the room. They shouted out seemingly random numbers.

Rahim: Like two circle, they draw people... This shake like [miming with self]

Nasir: Three

Rahim: Two. One time, with .. it will be 2. like if...

Rahim and Nasir talked with each other in Pashto.

Melinda: If there are sixteen people there will be 15 handshakes. It would be one handshake and two and three. [Gesturing as she counts up]

Nasir: Yeah, there is two...

Teacher: Did these people shake hands.

Rahim: Oh my god, you're right...

Hien: Four people would have six handshakes...

Osha: I thought you said 16

Hien: I changed my mind.

Teacher: What about for five people?

Students called out answers.

The teacher then asked students what they know or think about "systems."

Rahim: Different system...

Teacher: Organized...

Uzma: The cellphone has lots of systems... like a computer...

Teacher: Yes, like an organized order... How can we count five handshakes systematically...? Come up with a plan

The teacher directed students to act out the handshakes. Five students stood up (Melinda, Nasir, Rahim, Hien, and Uzma). They counted in different ways as Osha recorded. First, they shook hands “down the line” (i.e., Melinda shakes hands with everyone then steps off). Osha records this on the dry erase board (see Figure 1).

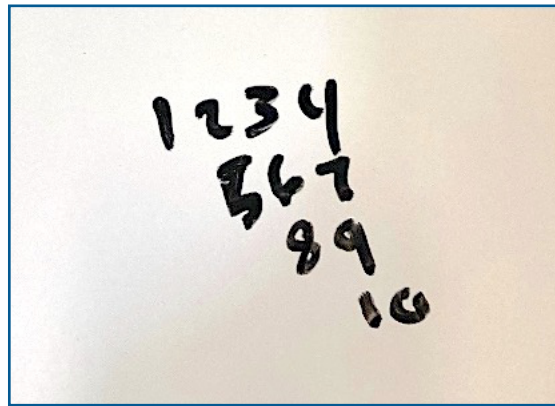


Figure 1. Osha Recorded the First Demonstration of Handshakes

The students then took another approach in a more circular arrangement. Nasir organized the others in a circle, where each shook hands with their immediate neighbor. Then, they skipped as they shook hands. That is, if the group members were A, B, C, D, and E, in order, it went rapidly as A with C, C with E, E with B, B with D, D with A. Osha recorded this demonstration on the board with a diagram (Figure 2).

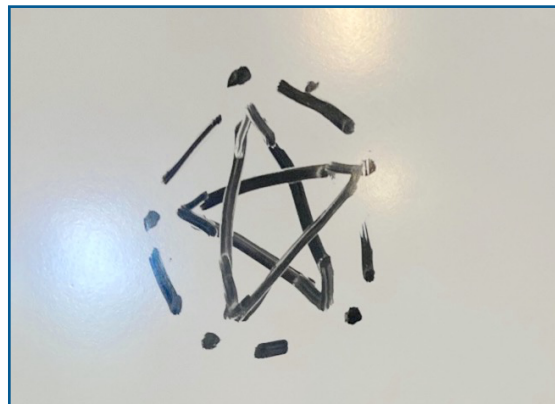


Figure 2. Osha’s Record of the Second, Circular Approach to Handshaking

The teacher had the students count up and confirm the number of handshakes among five people is ten handshakes, referring to either diagram.

Melinda: Then I need to find six people, seven people

The teacher wrote the sequence of the number of handshakes on the board [1, 3, 6, 10, ...] and asked students in their groups to extend to 10 people.

Nasir: No, they change the number, and have four in the middle... There is the missing... there is the 3 and 6, between the...

Teacher: Between the 10 we are missing what?

Rahim: I think the 10 people is 20 or something.

Teacher: Can you continue on your paper?

Rahim: The next one is 28...

Nasir: And the next one should be 36, next one is... 45

Melinda: You need to 36 plus 9.

Teacher: However, what was the question?

Nasir: There was like 55 hands or people... 55 is people or hands?

Teacher: [counting up to 10 people.] The first term is two people.... Counting up... 10th term.

Teacher: Can you think of a rule? For 150 kids, how many handshakes happened

Nasir: 55

Rahim: No, it's not 55.

Osha: It could be odd too.... The number they are using... So how many in the group might be, so do you go group by group...

Hien: But the groups have to find another group for that... we have to shake 11 times....

Osha: Then we don't count them.

Hien: You rotate ten times because there's ten people

Osha: Then...

Hien: Then you can ten, ten, ten

Osha: What if it's not even, what if it's odd?

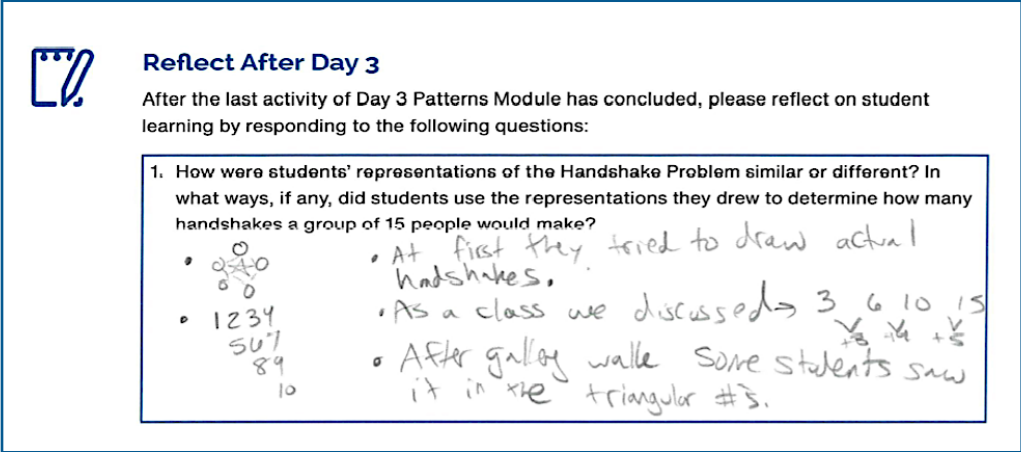
Hien: He said 150.

- Osha:** He said any, even any...
- Hien:** You can still do it...
- Osha:** With nine groups?
- Teacher:** Each change adds by one more. The handshake is adding by one.
- Hien:** It was adding on... like one more handshake....
- Teacher:** Maybe not one more handshake...
- Rahim:** No, not again.

The teacher asked them to think about the changes.

Osha talked about how where she is from (Nepal) and how they don't like shaking hands

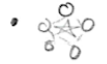
After this episode, the teacher wrote in his reflection journal (Figure 3).



Reflect After Day 3

After the last activity of Day 3 Patterns Module has concluded, please reflect on student learning by responding to the following questions:

1. How were students' representations of the Handshake Problem similar or different? In what ways, if any, did students use the representations they drew to determine how many handshakes a group of 15 people would make?

•  • At first they tried to draw actual handshakes.

• 1234
 567
 89
 10 • As a class we discussed $\rightarrow 3 \ 6 \ 10 \ 15$
 $\frac{1}{2} \times 14 + \frac{1}{2}$

• After gallery walk some students saw it in the triangular #'s.

Figure 3. Teacher Reflection on Student Approaches to Modeling of Handshakes